Putting Energy Back into Economics

Human society is energy blind. Like a fish in water, it takes for granted the existence of that without which it could not survive.

As with so many of humanity's problems, this conceptual failure can be traced back to an economist. However, the guilty party is not one of "the usual suspects"—Neoclassical economists—but the person virtually all economists describe as "the Father of Economics", Adam Smith.

Smith led economics astray on the vital issue of energy in the very first sentence of *The Wealth of Nations*, when he stated that:

> THE annual *labour* of every nation is the fund which originally supplies it with all the necessaries and conveniences of life which it annually consumes… (Smith 1776, p. 10. Emphasis added)

I emphasize "labour" in that sentence because, apart from that word, it is virtually identical to the opening sentence of Richard Cantillon's *Essay on Economic Theory*, which was published two decades before *The Wealth of Nations*:

> *Land* is the source or matter from which all wealth is drawn; man's labor provides the form for its production, and wealth in itself is nothing but the food, conveniences, and pleasures of life. (Cantillon 1755, p. 21. Emphasis added)

With that one word altered, economics took a terrible lurch away from realism and into fantasy. Cantillon's insight was that what existed before Man and outside human society—let alone outside "the economy"—was the source of the material wealth we generate within the economy. Smith's substitution saw an action within the economy itself—the work of the labourer—as the source of value, and the division of labour over time as the source of its growth.

Cantillon's perspective, that wealth originated outside the economy—though the form wealth took was shaped within it—was correct, according to the incontrovertible Laws of Thermodynamics (Ulgiati and Bianciardi 2004; Eddington 1928, p. 37). Smith's perspective was wrong, because he contemplated that the closed system of the economy could produce more outputs than inputs over time. This wasn't known to be false until a century after *The Wealth of Nations*, when the Laws of Thermodynamics were developed, so Smith cannot be criticised for that mistake. But economists today should not persist with models of production that violate the Laws of Thermodynamics.

From the First Law, that energy is conserved, we know that there cannot be a surplus of outputs over inputs. From the Second Law, that energy degrades when used to do work, we know that order declines over time in a closed system—which the economy, considered in isolation from the environment, is. So, even worse than "no surplus", there is "more disorder": the economy, considered in isolation from the environment, must degrade rather than grow.[1] To explain the economy, we must start from a flow of energy from the environment into the economy, and end with waste that must be dumped back into the environment, as a consequence, not merely of growth, but of any economic activity whatsoever, whether the economy is expanding or contracting.

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[1] Georgescu-Roegen gives a very accessible definition of both energy conservation and entropy: "In an isolated thermodynamic system the available energy continuously and irrevocably degrades into an equal quantity of unavailable energy, so that the total energy remains constant while the unavailable energy keeps increasing up to a maximum." (Georgescu-Roegen 1993, p. 187)

Classical and Neoclassical economics developed in ignorance of these Laws, and therefore developed in ignorance of the role of energy in production. Marshall used the term “energy” 79 times in the foundational text for Neoclassical economics, his *Principles of Economics* (Marshall 1890 [1920]), but always to describe human initiative and action, and not once in the thermodynamic sense. Energy, which should be front and centre in the economic analysis of production, instead disappeared from view.

Neoclassical Economics—the Cobb Douglas Production Function

Cobb and Douglas, when they developed the now dominant Neoclassical model of production, considered only Labour and Capital as inputs—though they did state that “we should ultimately look forward toward including the third factor of natural resources in our equations and of seeing to what degree this modifies our conclusions” (Cobb and Douglas 1928, p. 164). That was never done. Instead, after an initially rocky reception, the Cobb-Douglas Production Function (CDPF), with only Labour and Capital as inputs, became the accepted model of production for Neoclassical economists.[2] The reason for its acceptance was neatly expressed by Robert Solow when he quipped to Franklin Fisher that:

> had Douglas found labor's share to be 25 per cent and capital's 75 per cent instead of the other way around, we would not now be discussing aggregate production functions. (Fisher 1971, p. 305)

Cobb and Douglas found that result by fitting the function shown in Equation (1) to index number data, which they had laboriously assembled from Census data and an established index of manufacturing output (see Table 4 in the Appendix). In Equation (1), P stands for manufacturing output, L and C for employment and capital respectively in manufacturing, and b is a constant:

$$P = b \cdot L^{k} \cdot C^{1-k} \tag{1}$$

Their regression returned the result shown in Equation (2):

$$P^{'} = 1.01 \cdot L^{3/4} \cdot C^{1/4} \tag{2}$$

They reported an extremely high correlation coefficient, not merely for Equation (2), but for what they described as the data “with trends eliminated”:

> The coefficient of correlation between *P* and *P'* with trends included is .97 and with trends eliminated is .94. (Cobb and Douglas 1928, p. 154)

This implied a high level of robustness for their result, but this is not the case. The results and high correlations for the absolute value data are correct, but as Samuelson later observed, this was largely due to the collinearity of the data (Samuelson 1979, pp. 929). However, their stated results for the “trends eliminated” data are an artefact of their method of de-trending, which was to analyse the three-year moving average. When annual changes are used, the results are disastrous: the coefficient for α is negative (and, for what it’s worth, the R^2 is much lower)—see Table 1.

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[2] Computable General Equilibrium (CGE) models are an obvious exception, with their input-output tables for production, but over time these have become relics in Neoclassical modelling, with the Cobb-Douglas Production Function (in raw or CES—“Constant Elasticity of Substitution”—form) reigning supreme in the era of DSGE models.

Table 1: Parameter values and R^2 from the Cobb-Douglas index data, and annual fractional change in the data

Economists, data & assumptions	***Functions***	***Fitted values***	***R^2***
1. Cobb-Douglas original data	$P = b. C^{\alpha}.L^{1-\alpha}$	$b=1.02$, $\alpha = 0.25$	0.94
2. Cobb-Douglas change data	$\Delta P/P=\alpha.\Delta C/C+(1-\alpha).\Delta L/L$	$\alpha=-0.15$	0.66

The results are similarly bad when modern data is fitted—see Table 5 in the Appendix for the Penn World Tables data for the USA from 1950 till 2019 (Feenstra, Inklaar, and Timmer 2015) and the fractional annual rate of change. The results from fitting the CDPF to this data are shown in Table 2 and are similarly disastrous for Neoclassical theory. A fit of the CDPF to aggregate data returns an α of 1.24, which heavily weights Capital's contribution to output, and gives Labour a negative weight. The annual rates of change data generates a value for α which is less than 1, but also "wrong", in terms of the Neoclassical theory of income distribution: it attributes 71% of the change in output to Capital and only 29% to Labour. This may in fact be more realistic, but it conflicts with distribution of income data, and therefore with Neoclassical theory. As Solow said, had Cobb and Douglas returned results like these, Neoclassical economists "would not now be discussing aggregate production functions".

Table 2: CDPF fitted to PWT data for the USA from 1950 till 2019

Data	***Functions***	***Fitted values***	***R^2***
3. PWT rgdpna, emp & rnna	$P = b. C^{\alpha}.L^{1-\alpha}$	$b= 0.013$, $\alpha = 1.24$	0.997
4. Annual change fraction PWT	$\Delta P/P=\alpha.\Delta C/C+(1-\alpha).\Delta L/L$	$\alpha = 0.71$	0.29

Rescued by Solow's Residual

However, Neoclassical economists were saved the embarrassment of encountering these results by Solow's introduction of technical change into the CDPF. His intentions were laudable, but to achieve his objective he had to add two assumptions—that the exponents in the CDPF were the marginal products of Labour and Capital, and that these were equivalent to income-share data:

> The new wrinkle I want to describe is an elementary way of segregating variations in output per head due to technical change from those due to changes in the availability of capital per head. Naturally, every additional bit of information has its price. In this case the price consists of *one new required time series, the share of labor or property in total income*, and *one new assumption, that factors are paid their marginal products*. Since the former is probably more respectable than the other data I shall use, and since the latter is an assumption often made, the price may not be unreasonably high. (Solow 1957, p. 312. Emphasis added)

Of course, Neoclassical economists were more than willing to pay this "price", since it was to assume that their theory of production and of income distribution were both correct, and consistent with each other. They could then derive the contribution of change in technology from the difference between change in GDP and change in the two income-distribution-weighted "factors of production". From this date on, the exponents in the CDPF were not derived from empirical data, but were simply assumed to be correct, and equal to the shares of Labour and Capital in income distribution data—1/3rd for Capital and 2/3rds for Labour (Solow 1957, Table 1, p. 315). Variation between changes in output and the weighted changes in inputs was attributed to "total factor productivity" and measured by "the Solow Residual". The fact that, in Solow's initial paper, 87.5% of growth was attributed to technical change, and only 12.5% to changes in the factor proportions of Labour and Capital, was only moderately embarrassing. Subsequently, Neoclassical economists have since simply assumed that their models of production and distribution are correct, and the coefficients of the CDPF have altered from flawed empirical findings to unquestioned theoretical assumptions.

All of this was without considering energy: to this day, the vast majority of Neoclassical models of production consider only Labour and Capital as inputs. But when energy was considered by some Neoclassicals, it was accorded the same treatment: its exponent was set by its share in GDP, and this was assumed to be equal to its marginal productivity.

The Power(lessness) of Energy?

Two of the very few Neoclassical papers that include energy in a production function and ascribe a numerical value to it[3] are (Engström and Gars 2016) and (Bachmann et al. 2022). The former uses an exponent of 0.03 and the latter of 0.04, in production functions of the form shown in Equation (3):

$$F(K,\, L,\, E) \;=\; K^{\alpha} \cdot L^{1-\alpha-\nu} \cdot E^{\nu} \qquad (3)^4$$

Both made Solow's assumption that the share of energy in GDP is equal to the marginal productivity of Energy. This led Bachmann et al. to comment that:

> Therefore, for example, a drop in energy supply of *Δlog E = -10%* reduces production by *ΔlogY = 0.04 x 0.1 = 0.004 =0.4%* … [which] … "shows that *production is quite insensitive to energy E as expected*" (Bachmann et al. 2022, Appendix, p. 5. Emphasis added).

The data begs to differ. Table 6 and Figure 1 to Figure 5 show Gross World Product[5] against Primary Energy Supply[6] for the years 1971 till 2019. Far from production being "quite insensitive to energy", as assumed by Neoclassical economists, the empirically derived value of $\frac{\Delta Y / Y}{\Delta E / E}$ is 0.97, rather than the 0.03-0.04 value assumed by Neoclassical economists. Instead of production being "quite insensitive to energy", to a reasonable first approximation, production *is* Energy.

Table 3 shows the coefficients for regressing GDP and change in GDP |(ΔY/Y) against linear equations for Energy and change in Energy (ΔE/E).

Table 3: Regression of Energy against Gross World Product

Data	***Functions***	***Fitted values***	***R^2***
5. OECD Energy & World Bank GWP	P=a+b.E	a=3510, b = 0.14	0.99
6. Annual change fraction	ΔP/P=a+b. ΔE/E	a=-0.01, b = 0.97	0.7

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[3] (Solow 1974b; Solow 1974a; Stiglitz 1974b; Stiglitz 1974a) include energy or resources, but do not provide numerical values for the exponents.

[4] Equation 4c in (Engström and Gars 2016, p. 546)

[5] https://data.worldbank.org/indicator/NY.GDP.MKTP.CD.

[6] https://data.oecd.org/energy/primary-energy-supply.htm.

Figure 1: Gross World Product and Energy Consumption over time

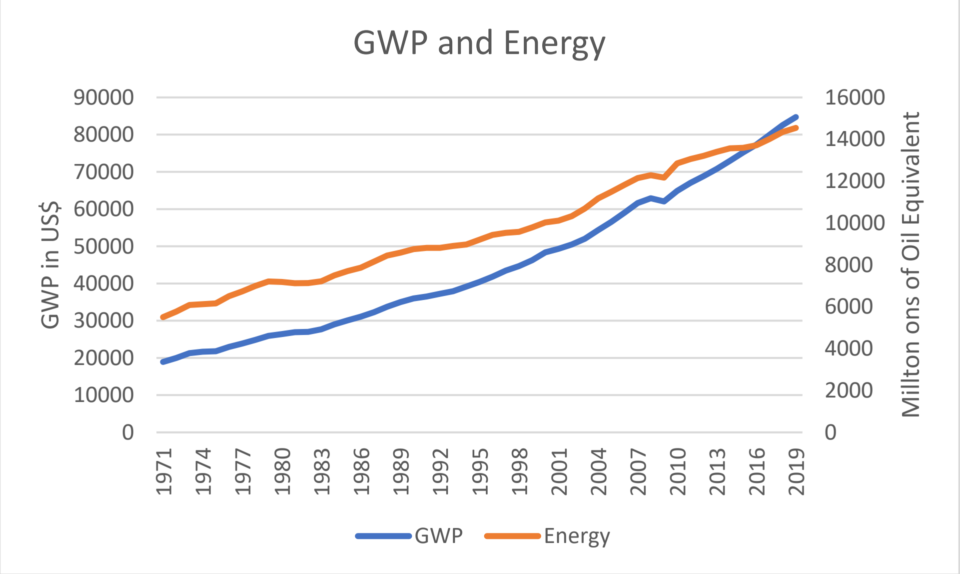


Figure 2: Energy vs GWP

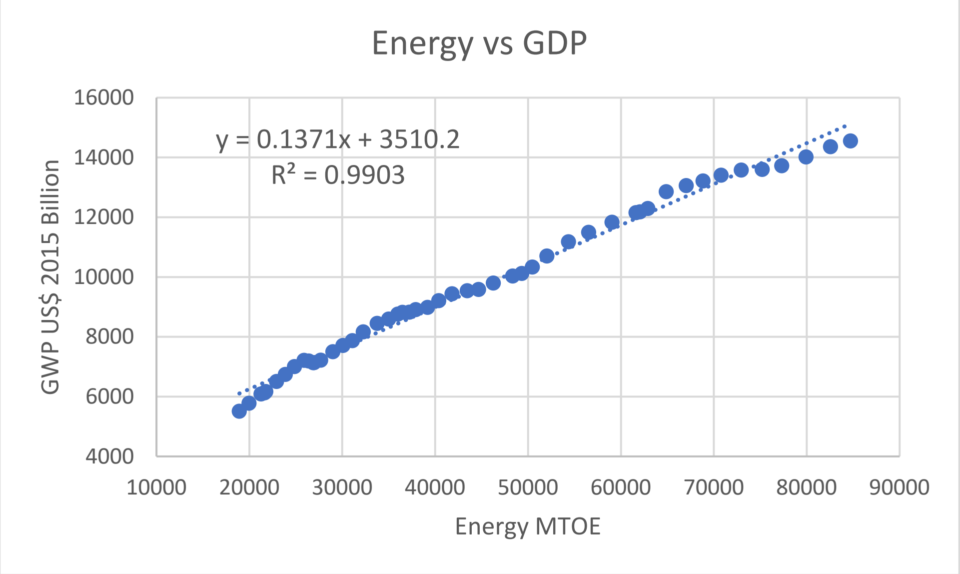


Figure 3: Ratio of GWP in US$2015 billion to Energy in MTOE from 1971 till 2019

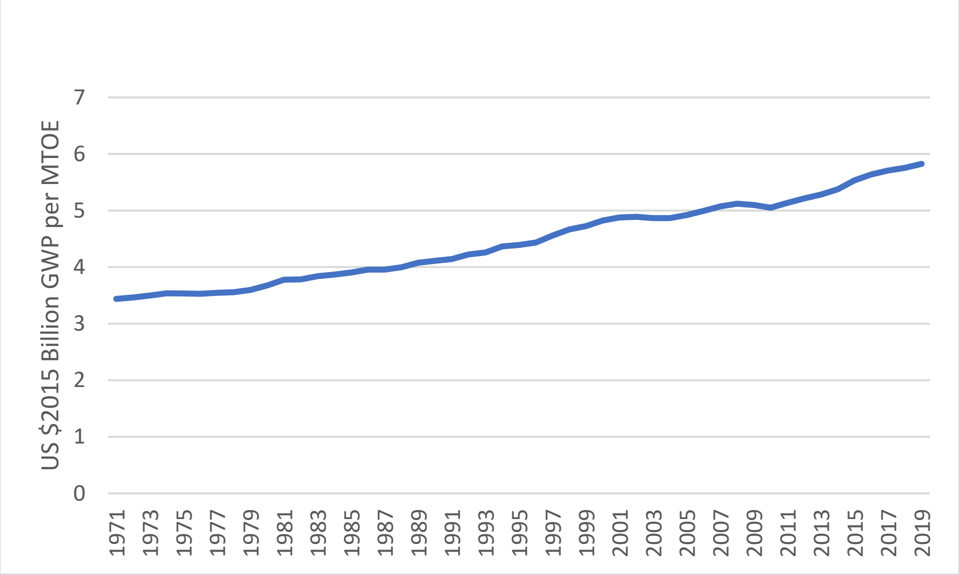


Figure 4: Change in GWP and Change in Energy in Percent p.a.

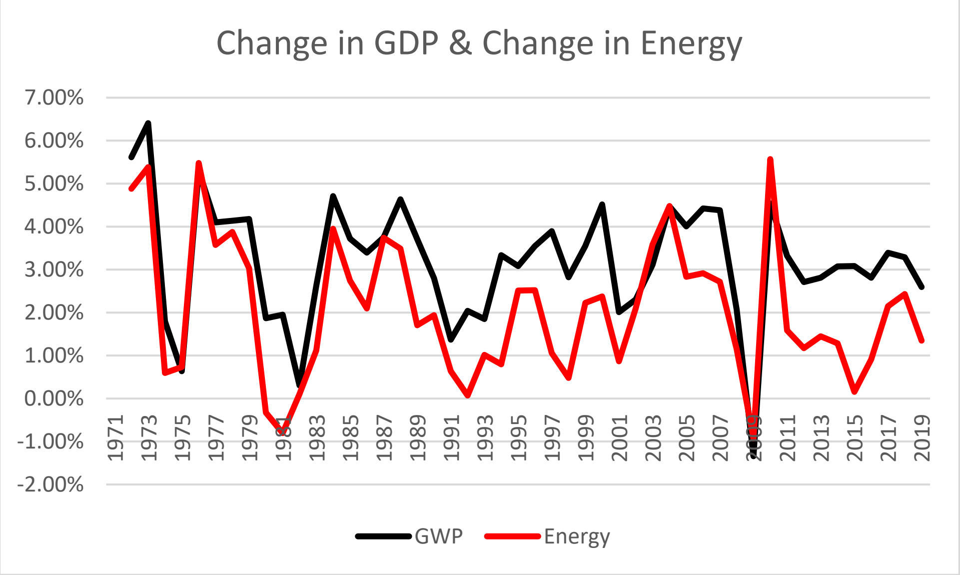
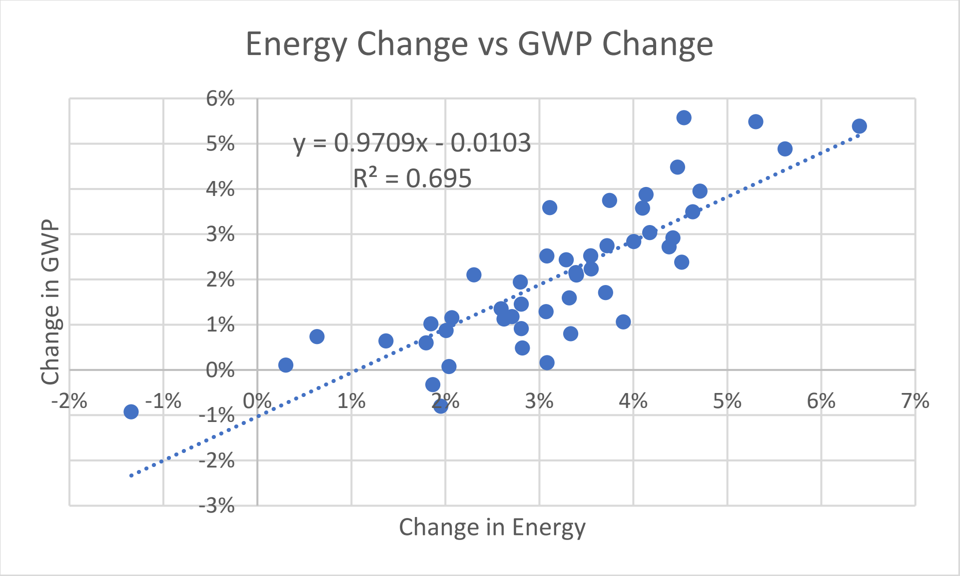


Figure 5: Correlation of change in Energy and change in GWP in Percent p.a.



This empirical data, as Bachmann et al. unintentionally show, is an effective refutation of the Neoclassical theories of production and income distribution, and confirmation of the Post-Keynesian theories.

They compare the polar opposites of the Cobb-Douglas and the Leontief in a CES production function, where the elasticity of substitution between inputs σ for Cobb-Douglas equals 1 and that for Leontief equals 0. They correctly lay out the implications of the Leontief case, that:

> Leontief production… implies that $Y = E/\alpha$ … and hence $\Delta \log Y = \Delta \log E$ … Therefore, if the elasticity of substitution is exactly zero, production Y drops one-for-one with energy supply E … *Intuitively, the Leontief assumption means that energy is an extreme bottleneck in production: when energy supply falls by 10%, the same fraction 10% of the other factors of production X lose all their value* (their marginal product drops to zero) *and hence production Y falls by 10%.* (Bachmann et al. 2022, Appendix, pp. 5-6. Emphasis added)

They plotted the *theoretical* relationships between energy input and GDP output for different values of the substitution parameter σ in their Figure 1 (reproduced as Figure 6 here).

Figure 6: Bachmann et al.'s theoretical predictions of change in output for a change in energy

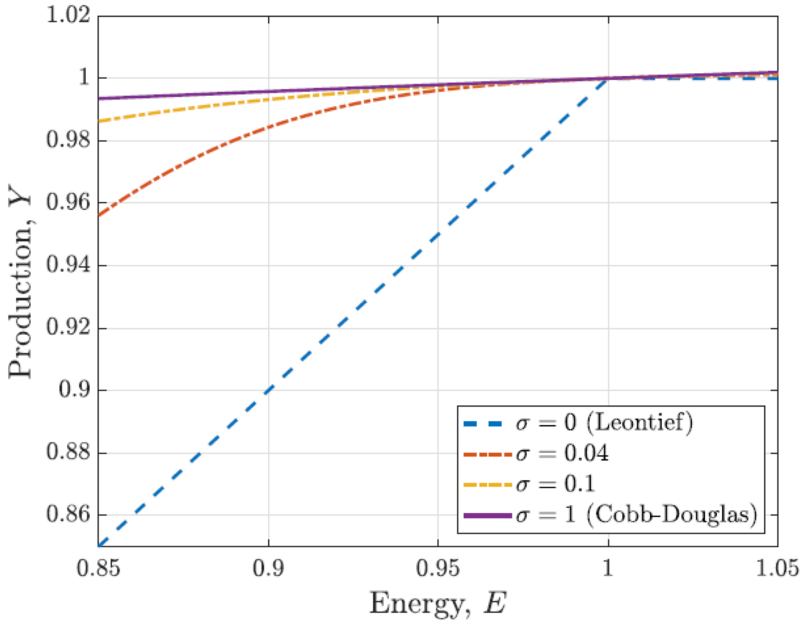


Figure 1: Output losses following a fall in energy supply for different elasticities of substitution

They then rejected the Leontief function, on the grounds that its prediction of a 1:1 fall in production for a fall in energy leads to nonsensical results *in terms of Neoclassical theory*:

> **Extreme scenarios with low elasticities of substitution and why Leontief production at the macro level is nonsensical** … The blue dashed line in Figure 1 showed that output falls one-for-one with energy supply in the Leontief case… *the marginal product of energy jumps to 1/α* [their exponent for energy] while *the marginal product of other factors … falls to zero*. ***If … factor prices equal marginal products***, this then implies that similarly the price of energy jumps to 1/α and the prices of other factors a fall to zero… this then also implies that the expenditure share on energy jumps to 100% whereas the expenditure share on other factors falls to 0%. *We consider these predictions to be economically nonsensical*. (Bachmann et al. 2022, p. 15. Italicised emphasis added)

These predictions *are* nonsensical, but at the same time, *the Leontief case fits the empirical data* (which, following Solow's lead, they did not consult). It is not the data which is false, but the assumption they made that "factor prices equal marginal products". Therefore, wages, profits and the price of energy cannot be based upon the "marginal product" of labour, capital and energy respectively. The Neoclassical Cobb-Douglas model of production is false,[7] and the Post-Keynesian Leontief model of production is correct. The question now remains as to why the Leontief model is correct.

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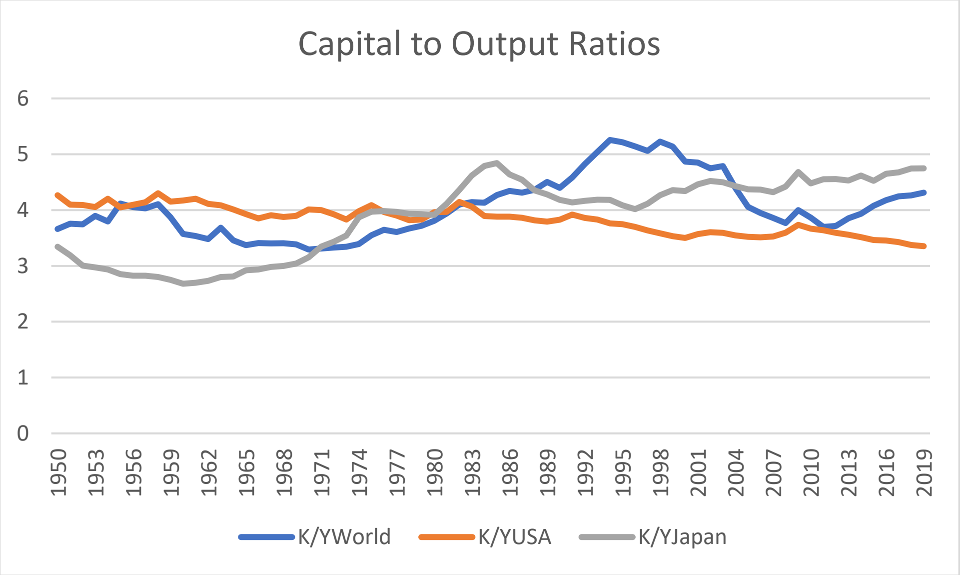
[7] This empirical critique adds to the logical critique made by Shaikh (Shaikh 2005, 1987, 1980, 1974) and McCombie and Felipe (Felipe and McCombie 2020; Felipe and McCombie 2014; Felipe and McCombie 2011; Felipe and McCombie 2007; McCombie 2000) that the Cobb-Douglas function is simply a nonlinear mapping from the income identity that Wages plus Profits equals Income, given the empirically realistic assumption of relatively slow changes in the distribution of income.

From Empirical Regularity to the Role of Energy in Production

The Leontief Production Function began as an empirical regularity between GDP, however measured, and Capital, however measured. The ratio was relatively constant over time and showed no trend—see Figure 7 for Capital to Output ratios derived from the Penn World Tables database.

Figure 7: Capital to Output Ratios are reasonably constant over time



This led to the pragmatic Post-Keynesian school adopting the capital to output ratio as its "production function", with the justification that this relationship was found in the data, but with no real explanation as to *why* it was found. Leaving aside the minimum form in which the LPF is often expressed but seldom used, we have, as in the Goodwin model (Goodwin 1967):

$$Y = \frac{K}{v} \tag{4}$$

Here *v* is the capital to output ratio. With *K* having the dimension of Widgets, and *Y* of Widgets per year, for dimensional accuracy, *v* must be a time constant denominated in Years.

The empirical regularity behind the LPF can be explained by the aphorism that Ayres, Standish and I applied in "A Note on the Role of Energy in Production" (Keen, Ayres, and Standish 2019), that:

> labour without energy is a corpse, while capital without energy is a sculpture. (Keen, Ayres, and Standish 2019):

This suggested that the inputs of Labour and Capital assumed by both the *CDPF* and the *LPF* should be replaced by the energy inputs to both Labour and Capital, via the substitution shown in Equation (5):

$$\begin{aligned} L &\rightarrow L \cdot E_L \cdot e_L \\ K &\rightarrow K \cdot E_K \cdot e_K \end{aligned} \tag{5}$$

Here respectively L and K stand for units of Labour and Capital,[8] E_L and E_K represent the energy consumed by a unit of Labour and a unit of Capital, and e_L and e_K are time constants (dimensioned by 1/Year) representing the proportion of these inputs that are turned into useful work over a Year. This then suggests a way to derive the *LPF* from the dimensionality of the substitution proposed in Equation (5). In the standard single commodity *CDPF* and *LPF*, Y and K are denominated in "widgets per year" and "widgets" respectively—units of a universal commodity that can be used for either investment or consumption:

$$Y \Rightarrow \frac{Widget}{Year} \tag{6}$$

The substitution in (5) on the other hand has the dimensionality of units of Energy per year:

$$K \cdot E_K \cdot e_K \Rightarrow Widget \cdot \frac{Energy}{Widget} \cdot \frac{Scalar}{Year} \Rightarrow \frac{Energy}{Year} \cdot Scalar \tag{7}$$

Call this Q, denominated in units of Energy per year, to distinguish it from Y, denominated in units of widgets per year:

$$Q = K \cdot E_K \cdot e_K \tag{8}$$

Y is therefore equal to Q divided by E_K:

$$\begin{aligned} Y &= \frac{Q}{E_K} \\ &= \frac{K \cdot E_K \cdot e_K}{E_K} \\ &= K \cdot e_K \end{aligned} \tag{9}$$

Equating Equation (9) with Equation (4) shows that the empirically derived capital to output ratio v is in fact the inverse of the proportion of inputted energy that machinery turns into useful work:

$$\begin{aligned} \frac{K}{v} &= K \cdot e_K \\ e_K &= \frac{1}{v} \end{aligned} \tag{10}$$

This provides a physical explanation for the empirical regularity on which the Post-Keynesian model of production is based: it is due to the role of machinery in turning energy—predominantly fossil fuel energy—into useful work. This model therefore ties Post-Keynesian theory to the initial accurate insights of the Physiocrats, that Nature is the source of wealth, and that what human ingenuity does is enable the conversion of "this superfluity that nature accords him as a pure gift" (Turgot 1774, p. 9) into useful work. Given the close relationship between GDP and Energy shown in Figure 2 to Figure 5, at a first approximation, GDP *is* useful energy. Equation (9) can therefore be used in place of Equation (4) in Post-Keynesian models.

The Post-Keynesian model is also consistent with the Laws of Thermodynamics, including the Second Law (which the Physiocrats did not realise) that doing work generates waste as well as desired

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[8] Units of Capital raise all the issues in the Capital Controversies (McCombie 2001; Harcourt 1972), but this formulation also enables an empirically sound way around them.

output. With the capital to output ratio averaging 4 globally, and ranging between 3 and 5 for developed nations, the magnitude of e_K is of the order of 0.2-0.33. This then quantifies the waste generated in production as being of the order of 0.67-0.8: humanity generates more waste than output. The constancy of the capital to output ratio, much criticised by Neoclassical economists, is in fact due to the impossibility of substituting any other input for energy, and intrinsic limits to the efficiency of conversion of energy into useful work given by the Second Law of Thermodynamics.[9]

Conclusion

The Neoclassical Cobb-Douglas Production Function, with its exponents assumed to be equal to the income shares of factor inputs, and also equal to the marginal product of those inputs, cannot be reconciled with energy data, or with the Laws of Thermodynamics, and it is therefore wrong.

The Post-Keynesian Leontief Production Function, on the other hand, is not only empirically accurate, but is also consistent with the Laws of Thermodynamics. Though the construction of a universal commodity in aggregate production functions has always been a convenience, the fact that GDP and Energy are so tightly coupled means that the LPF is a reasonable first approximation to reality. Solow's observation that "As long as we insist on practicing macroeconomics we shall need aggregate relationships" (Solow 1957, p. 213) is correct, but the only aggregate production function that fits the bill is the Leontief Production Function.

Appendix: Data Tables

Table 4: Cobb-Douglas Data set in Tables II, III & IV (Cobb and Douglas 1928, pp. 145, 148, 149) and annual rates of change

Cobb-Douglas Index Numbers							
Year	GDP	Labour	Capital	Annual Change Fraction			
1899	100	100	100	Year	GDP	Labour	Capital
1900	101	105	107	1900	0.010000	0.050000	0.070000
1901	112	110	114	1901	0.108911	0.047619	0.065421
1902	122	118	122	1902	0.089286	0.072727	0.070175
1903	124	123	131	1903	0.016393	0.042373	0.073770
1904	122	116	138	1904	-0.016129	-0.056911	0.053435
1905	143	125	149	1905	0.172131	0.077586	0.079710
1906	152	133	163	1906	0.062937	0.064000	0.093960
1907	151	138	176	1907	-0.006579	0.037594	0.079755
1908	126	121	185	1908	-0.165563	-0.123188	0.051136
1909	155	140	198	1909	0.230159	0.157025	0.070270
1910	159	144	208	1910	0.025806	0.028571	0.050505
1911	153	145	216	1911	-0.037736	0.006944	0.038462
1912	177	152	226	1912	0.156863	0.048276	0.046296
1913	184	154	236	1913	0.039548	0.013158	0.044248
1914	169	149	244	1914	-0.081522	-0.032468	0.033898
1915	189	154	266	1915	0.118343	0.033557	0.090164

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[9] "It would be a mistake to think that the limits imposed by the conversion of heat to mechanical work are of a technical nature (i.e., are due to the unavailability of tools or cylinders and pistons without friction or perfectly sealed)… *it is a constraint imposed by Nature and not by the use of technically poor instruments*." (Ulgiati and Bianciardi 2004, pp. 112-13. Emphasis added).

1916	225	182	298	1916	0.190476	0.181818	0.120301
1917	227	196	335	1917	0.008889	0.076923	0.124161
1918	223	200	366	1918	-0.017621	0.020408	0.092537
1919	218	193	387	1919	-0.022422	-0.035000	0.057377
1920	231	193	407	1920	0.059633	0.000000	0.051680
1921	179	147	417	1921	-0.225108	-0.238342	0.024570
1922	240	161	431	1922	0.340782	0.095238	0.033573

Table 5: Penn World Tables USA data (rgdpna, emp & rnna)

	Penn World Tables Data					
Year	Output	Labour	Capital	Annual Change Fraction		
1950	2466600	63	10563300	Output	Labour	Capital
1951	2665370	65	10898400	0.08058	0.03175	0.03172
1952	2773900	66	11258900	0.04072	0.01538	0.03308
1953	2904120	67	11667800	0.04694	0.01515	0.03632
1954	2887740	66	12058000	-0.00564	-0.01493	0.03344
1955	3093410	67	12496100	0.07122	0.01515	0.03633
1956	3159360	69	12930100	0.02132	0.02985	0.03473
1957	3225890	69	13353800	0.02106	0.00000	0.03277
1958	3202170	68	13737200	-0.00735	-0.01449	0.02871
1959	3423190	70	14221700	0.06902	0.02941	0.03527
1960	3510940	71	14693100	0.02563	0.01429	0.03315
1961	3600620	71	15185900	0.02554	0.00000	0.03354
1962	3820850	72	15739600	0.06116	0.01408	0.03646
1963	3987210	73	16341000	0.04354	0.01389	0.03821
1964	4217160	75	17002600	0.05767	0.02740	0.04049
1965	4491260	77	17730600	0.06500	0.02667	0.04282
1966	4787380	79	18508200	0.06593	0.02597	0.04386
1967	4918730	81	19239900	0.02744	0.02532	0.03953
1968	5160200	82	19990600	0.04909	0.01235	0.03902
1969	5322270	84	20734700	0.03141	0.02439	0.03722
1970	5333000	85	21379000	0.00202	0.01190	0.03107
1971	5508630	85	22033200	0.03293	0.00000	0.03060
1972	5798320	87	22780100	0.05259	0.02353	0.03390
1973	6125680	90	23606600	0.05646	0.03448	0.03628
1974	6092570	91	24310800	-0.00541	0.01111	0.02983
1975	6080050	90	24845800	-0.00205	-0.01099	0.02201
1976	6407650	93	25479500	0.05388	0.03333	0.02551
1977	6703950	96	26253300	0.04624	0.03226	0.03037
1978	7075040	100	27174900	0.05535	0.04167	0.03510
1979	7299040	103	28144200	0.03166	0.03000	0.03567
1980	7280300	103	28952200	-0.00257	0.00000	0.02871
1981	7465060	104	29738300	0.02538	0.00971	0.02715
1982	7330470	103	30368400	-0.01803	-0.00962	0.02119

1983	7666490	105	31093100	0.04584	0.01942	0.02386
1984	8221290	109	32068500	0.07237	0.03810	0.03137
1985	8564090	111	33127000	0.04170	0.01835	0.03301
1986	8860630	114	34182000	0.03463	0.02703	0.03185
1987	9167170	117	35208600	0.03460	0.02632	0.03003
1988	9550090	119	36224200	0.04177	0.01709	0.02885
1989	9900830	122	37242100	0.03673	0.02521	0.02810
1990	10087600	123	38181100	0.01886	0.00820	0.02521
1991	10076600	122	38943600	-0.00109	-0.00813	0.01997
1992	10431600	122	39749600	0.03523	0.00000	0.02070
1993	10718700	123	40619400	0.02752	0.00820	0.02188
1994	11150600	126	41576900	0.04029	0.02439	0.02357
1995	11449900	127	42589100	0.02684	0.00794	0.02435
1996	11881800	129	43715500	0.03772	0.01575	0.02645
1997	12410300	132	44934900	0.04448	0.02326	0.02789
1998	12966400	135	46303800	0.04481	0.02273	0.03046
1999	13582700	137	47787900	0.04753	0.01481	0.03205
2000	14143400	139	49329500	0.04128	0.01460	0.03226
2001	14284600	139	50697100	0.00998	0.00000	0.02772
2002	14533400	138	51889200	0.01742	-0.00719	0.02351
2003	14949200	139	53148100	0.02861	0.00725	0.02426
2004	15517100	140	54536800	0.03799	0.00719	0.02613
2005	16062200	142	56048800	0.03513	0.01429	0.02772
2006	16520800	145	57539800	0.02855	0.02113	0.02660
2007	16830800	146	58882200	0.01876	0.00690	0.02333
2008	16807800	146	59952000	-0.00137	0.00000	0.01817
2009	16381400	141	60486900	-0.02537	-0.03425	0.00892
2010	16801400	141	61035300	0.02564	0.00000	0.00907
2011	17062000	142	61657100	0.01551	0.00709	0.01019
2012	17445800	145	62424400	0.02249	0.02113	0.01244
2013	17767100	146	63225100	0.01842	0.00690	0.01283
2014	18215900	148	64118500	0.02526	0.01370	0.01413
2015	18776200	150	65053600	0.03076	0.01351	0.01458
2016	19097500	152	65971300	0.01711	0.01333	0.01411
2017	19543000	155	66940300	0.02333	0.01974	0.01469
2018	20128600	157	68005600	0.02996	0.01290	0.01591
2019	20563600	158	69059500	0.02161	0.00637	0.01550

Table 6: GWP (US$ 2015 Billion) and Primary Energy (Million Tons of Oil Equivalent)

	US2017$ Bn	MTOE	Ratio of GWP to Energy	Change Fraction	
Year	GWP	Energy		GWP	Energy
1971	18925	5504	3.44		
1972	19988	5773	3.46	0.0562	0.0488

1973	21269	6084	3.50	0.0641	0.0538
1974	21651	6120	3.54	0.0180	0.0060
1975	21789	6165	3.53	0.0064	0.0073
1976	22944	6502	3.53	0.0530	0.0548
1977	23885	6735	3.55	0.0410	0.0357
1978	24873	6996	3.56	0.0414	0.0387
1979	25912	7208	3.60	0.0418	0.0303
1980	26396	7184	3.67	0.0187	-0.0033
1981	26911	7126	3.78	0.0195	-0.0081
1982	26993	7133	3.78	0.0030	0.0010
1983	27701	7213	3.84	0.0262	0.0112
1984	29006	7498	3.87	0.0471	0.0395
1985	30085	7704	3.91	0.0372	0.0274
1986	31107	7865	3.96	0.0340	0.0210
1987	32272	8159	3.96	0.0375	0.0374
1988	33768	8444	4.00	0.0463	0.0349
1989	35019	8588	4.08	0.0371	0.0170
1990	36000	8754	4.11	0.0280	0.0194
1991	36493	8810	4.14	0.0137	0.0064
1992	37237	8816	4.22	0.0204	0.0007
1993	37925	8906	4.26	0.0185	0.0101
1994	39190	8976	4.37	0.0333	0.0079
1995	40398	9202	4.39	0.0308	0.0251
1996	41831	9434	4.43	0.0355	0.0252
1997	43460	9534	4.56	0.0389	0.0106
1998	44686	9580	4.66	0.0282	0.0048
1999	46274	9793	4.73	0.0355	0.0223
2000	48363	10026	4.82	0.0452	0.0238
2001	49335	10113	4.88	0.0201	0.0087
2002	50472	10325	4.89	0.0230	0.0210
2003	52042	10695	4.87	0.0311	0.0358
2004	54370	11173	4.87	0.0447	0.0448
2005	56547	11490	4.92	0.0400	0.0283
2006	59047	11825	4.99	0.0442	0.0291
2007	61634	12146	5.07	0.0438	0.0272
2008	62910	12286	5.12	0.0207	0.0115
2009	62066	12172	5.10	-0.0134	-0.0093
2010	64884	12850	5.05	0.0454	0.0557
2011	67039	13054	5.14	0.0332	0.0159
2012	68856	13207	5.21	0.0271	0.0117
2013	70790	13398	5.28	0.0281	0.0145
2014	72966	13570	5.38	0.0307	0.0128
2015	75215	13591	5.53	0.0308	0.0015
2016	77328	13715	5.64	0.0281	0.0091

2017	79949	14010	5.71	0.0339	0.0215
2018	82578	14350	5.75	0.0329	0.0243
2019	84720	14544	5.83	0.0259	0.0135

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